# WHAT IS A LOCK-IN AMPLIFIER?

Lock-in amplifiers are used to detect and measure very small AC signals - all the way down to a few nanovolts! Accurate measurements may be made even when the small signal is obscured by noise sources many thousands of times larger.

Lock-in amplifiers use a technique known as phase-sensitive detection to single out the component of the signal at a specific reference frequency AND phase. Noise signals at frequencies other than the reference frequency are rejected and do not affect the measurement.

## Why use a lock-in?

Let's consider an example. Suppose the signal is a 10 nV sine wave at 10 kHz. Clearly some amplification is required. A good low noise amplifier will have about 5 nV/√Hz of input noise. If the amplifier bandwidth is 100 kHz and the gain is 1000, then we can expect our output to be 10 µV of signal (10 nV x 1000) and 1.6 mV of broadband noise (5 nV/√Hz x √100 kHz x 1000). We won't have much luck measuring the output signal unless we single out the frequency of interest.

If we follow the amplifier with a band pass filter with a Q=100 (a VERY good filter) centered at 10 kHz, any signal in a 100 Hz bandwidth will be detected (10 kHz/Q). The noise in the filter pass band will be 50 µV (5 nV/√Hz x √100 Hz x 1000) and the signal will still be 10 µV. The output noise is much greater than the signal and an accurate measurement can not be made. Further gain will not help the signal to noise problem.

Now try following the amplifier with a phase-sensitive detector (PSD). The PSD can detect the signal at 10 kHz with a bandwidth as narrow as 0.01 Hz! In this case, the noise in the detection bandwidth will be only 0.5 µV (5 nV/√Hz x √.01 Hz x 1000) while the signal is still 10 µV. The signal to noise ratio is now 20 and an accurate measurement of the signal is possible.

## What is phase-sensitive detection?

Lock-in measurements require a frequency reference. Typically an experiment is excited at a fixed frequency (from an oscillator or function generator) and the lock-in detects the response from the experiment at the reference frequency. In the diagram below, the reference signal is a square wave at frequency $\omega_r$. This might be the sync output from a function generator. If the sine output from the function generator is used to excite the experiment, the response might be the signal waveform shown below. The signal is $V_{sig}\sin(\omega_r t + \theta_{sig})$ where $V_{sig}$ is the signal amplitude.

The SR830 generates its own sine wave, shown as the lock-in reference below. The lock-in reference is $V_L\sin(\omega_L t + \theta_{ref})$.
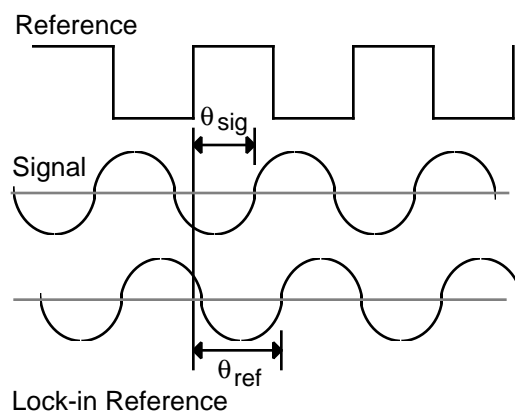


The SR830 amplifies the signal and then multiplies it by the lock-in reference using a phase-sensitive detector or multiplier. The output of the PSD is simply the product of two sine waves.

$$V_{psd} = V_{sig}V_L\sin(\omega_r t + \theta_{sig})\sin(\omega_L t + \theta_{ref})$$

$$= 1/2\, V_{sig}V_L\cos([\omega_r - \omega_L]t + \theta_{sig} - \theta_{ref}) - 1/2\, V_{sig}V_L\cos([\omega_r + \omega_L]t + \theta_{sig} + \theta_{ref})$$

The PSD output is two AC signals, one at the difference frequency ($\omega_r - \omega_L$) and the other at the sum frequency ($\omega_r + \omega_L$).

If the PSD output is passed through a low pass filter, the AC signals are removed. What will be left? In the general case, nothing. However, if $\omega_r$ equals $\omega_L$, the difference frequency component will be a DC signal. In this case, the filtered PSD output will be

$$V_{psd} = 1/2\, V_{sig}V_L\cos(\theta_{sig} - \theta_{ref})$$

This is a very nice signal - it is a DC signal proportional to the signal amplitude.

## Narrow band detection

Now suppose the input is made up of signal plus noise. The PSD and low pass filter only detect signals whose frequencies are very close to the lock-in reference frequency. Noise signals at frequencies far from the reference are attenuated at the PSD output by the low pass filter (neither $\omega_{noise}-\omega_{ref}$ nor $\omega_{noise}+\omega_{ref}$ are close to DC). Noise at frequencies very close to the reference frequency will result in very low frequency AC outputs from the PSD ($|\omega_{noise}-\omega_{ref}|$ is small). Their attenuation depends upon the low pass filter bandwidth and roll-off. A narrower bandwidth will remove noise sources very close to the reference frequency, a wider bandwidth allows these signals to pass. The low pass filter bandwidth determines the bandwidth of detection. Only the signal at the reference frequency will result in a true DC output and be unaffected by the low pass filter. This is the signal we want to measure.

## Where does the lock-in reference come from?

We need to make the lock-in reference the same as the signal frequency, i.e. $\omega_r = \omega_L$. Not only do the frequencies have to be the same, the phase between the signals can not change with time, otherwise $\cos(\theta_{sig} - \theta_{ref})$ will change and $V_{psd}$ will not be a DC signal. In other words, the lock-in reference needs to be phase-locked to the signal reference.

Lock-in amplifiers use a phase-locked-loop (PLL) to generate the reference signal. An external reference signal (in this case, the reference square wave) is provided to the lock-in. The PLL in the lock-in locks the internal reference oscillator to this external reference, resulting in a reference sine wave at $\omega_r$ with a fixed phase shift of $\theta_{ref}$. Since the PLL actively tracks the external reference, changes in the external reference frequency do not affect the measurement.

## All lock-in measurements require a reference signal.

In this case, the reference is provided by the excitation source (the function generator). This is called an external reference source. In many situations, the SR830's internal oscillator may be used instead. The internal oscillator is just like a function generator (with variable sine output and a TTL sync) which is always phase-locked to the reference oscillator.

## Magnitude and phase

Remember that the PSD output is proportional to $V_{sig}\cos\theta$ where $\theta = (\theta_{sig} - \theta_{ref})$. $\theta$ is the phase difference between the signal and the lock-in reference oscillator. By adjusting $\theta_{ref}$ we can make $\theta$ equal to zero, in which case we can measure $V_{sig}$ ($\cos\theta=1$). Conversely, if $\theta$ is 90°, there will be no output at all. A lock-in with a single PSD is called a single-phase lock-in and its output is $V_{sig}\cos\theta$.

This phase dependency can be eliminated by adding a second PSD. If the second PSD multiplies the signal with the reference oscillator shifted by 90°, i.e. $V_L\sin(\omega_L t + \theta_{ref} + 90°)$, its low pass filtered output will be

$$V_{psd2} = 1/2\ V_{sig}V_L\sin(\theta_{sig} - \theta_{ref})$$

$$V_{psd2} \sim V_{sig}\sin\theta$$

Now we have two outputs, one proportional to $\cos\theta$ and the other proportional to $\sin\theta$. If we call the first output X and the second Y,

$$X = V_{sig}\cos\theta \qquad Y = V_{sig}\sin\theta$$

these two quantities represent the signal as a vector relative to the lock-in reference oscillator. X is called the 'in-phase' component and Y the 'quadrature' component. This is because when $\theta=0$, X measures the signal while Y is zero.

By computing the magnitude (R) of the signal vector, the phase dependency is removed.

$$R = (X^2 + Y^2)^{1/2} = V_{sig}$$

R measures the signal amplitude and does not depend upon the phase between the signal and lock-in reference.

A dual-phase lock-in, such as the SR830, has two PSD's, with reference oscillators 90° apart, and can measure X, Y and R directly. In addition, the phase $\theta$ between the signal and lock-in reference, can be measured according to

$$\theta = \tan^{-1}(Y/X)$$

# WHAT DOES A LOCK-IN MEASURE?

So what exactly does the SR830 measure? Fourier's theorem basically states that any input signal can be represented as the sum of many, many sine waves of differing amplitudes, frequencies and phases. This is generally considered as representing the signal in the "frequency domain". Normal oscilloscopes display the signal in the "time domain". Except in the case of clean sine waves, the time domain representation does not convey very much information about the various frequencies which make up the signal.

## What does the SR830 measure?

The SR830 multiplies the signal by a pure sine wave at the reference frequency. All components of the input signal are multiplied by the reference simultaneously. Mathematically speaking, sine waves of differing frequencies are orthogonal, i.e. the average of the product of two sine waves is zero unless the frequencies are EXACTLY the same. In the SR830, the product of this multiplication yields a DC output signal proportional to the component of the signal whose frequency is exactly locked to the reference frequency. The low pass filter which follows the multiplier provides the averaging which removes the products of the reference with components at all other frequencies.

The SR830, because it multiplies the signal with a pure sine wave, measures the single Fourier (sine) component of the signal at the reference frequency. Let's take a look at an example. Suppose the input signal is a simple square wave at frequency f. The square wave is actually composed of many sine waves at multiples of f with carefully related amplitudes and phases. A 2V pk-pk square wave can be expressed as

$$S(t) = 1.273\sin(\omega t) + 0.4244\sin(3\omega t) + 0.2546\sin(5\omega t) + ...$$

where $\omega = 2\pi f$. The SR830, locked to f will single out the first component. The measured signal will be $1.273\sin(\omega t)$, not the 2V pk-pk that you'd measure on a scope.

In the general case, the input consists of signal plus noise. Noise is represented as varying signals at all frequencies. The ideal lock-in only responds to noise at the reference frequency. Noise at other frequencies is removed by the low pass filter following the multiplier. This "bandwidth narrowing" is the primary advantage that a lock-in amplifier provides. Only inputs at frequencies at the reference frequency result in an output.

## RMS or Peak?

Lock-in amplifiers as a general rule display the input signal in Volts RMS. When the SR830 displays a magnitude of 1V (rms), the component of the input signal at the reference frequency is a sine wave with an amplitude of 1 Vrms or 2.8 V pk-pk.

Thus, in the previous example with a 2 V pk-pk square wave input, the SR830 would detect the first sine component, $1.273\sin(\omega t)$. The measured and displayed magnitude would be 0.90 V (rms) ($1/\sqrt{2}$ x 1.273).

## Degrees or Radians?

In this discussion, frequencies have been referred to as f (Hz) and $\omega$ ($2\pi f$ radians/sec). This is because people measure frequencies in cycles per second and math works best in radians. For purposes of measurement, frequencies as measured in a lock-in amplifier are in Hz. The equations used to explain the actual calculations are sometimes written using $\omega$ to simplify the expressions.

Phase is always reported in degrees. Once again, this is more by custom than by choice. Equations written as $\sin(\omega t + \theta)$ are written as if $\theta$ is in radians mostly for simplicity. Lock-in amplifiers always manipulate and measure phase in degrees.

# THE FUNCTIONAL SR830

The functional block diagram of the SR830 DSP Lock-In Amplifier is shown below. The functions in the gray area are handled by the digital signal processor (DSP). We'll discuss the DSP aspects of the SR830 as they come up in each functional block description.



**SR830 FUNCTIONAL BLOCK DIAGRAM**